FUNCTION

Let A and B are two non-empty sets then a relation from set A to B is said to be a function if every element/member of set A has unique image in set B.
A function is also called **Mapping, Transformation** and **operator**.

If f is a Function from X to Y then

- ✓ For each element $x \in X, \exists\ y \in Y$.
  - y : Image of x
  - x : Pre image of y
- ✓ Different element of *x* may be associated with the same element of *y*.
- ✓ There may exist some element in *y* which are not associated with any element of *x*.

Domain, Codomain and Range:
Let f is a function such that f: A ⟶B. Set A is called domain, Set B is called Co-domain & element of B which are images of element of set A is called range.

TYPE OF FUNCTION

Injective or One to One Function: A function f: A ⟶B is called injection or one to one function if different element in the domain A have distinct (different) Images.

i.e. f is one to one.

If $f(a) = (a')$

$a = a'$

Many to One: A function f: A ⟶B is called many to one function if different element in the domain A have not distinct (different) Images.

$$\text{If } f(a) = f(b)$$
$$a \neq b$$

Surjection or Into Function : A function f: A ⟶B is said to be an onto function if each element of B is the image of some element.
In other word range cover domain completely i.e. if $f(A) = B$.

Into Function :- A function which is not surjection is called 'into' function.
i.e. f: A ⟶B

$f(A) \neq B$

Bijection or One-One Onto Function:-
Let f: A ⟶B. If it is an injection as well as surjection is known as bijection.

Inverse of A Function :

- ✓ Let f: A ⟶B f is bijection mapping from A to B then f^{-1}={(*y,x*), (*x,y*) ∈ *f*} is called inverse .
- ✓ A function f: A ⟶B is invertible function if and only if f: A ⟶B is bijection.

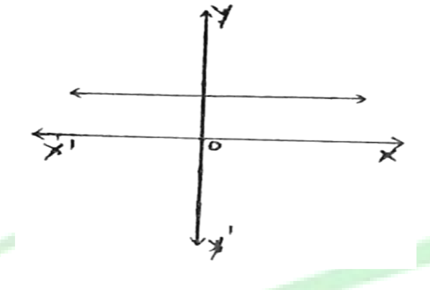
COMPOSITION OF FUNCTION

Let *f* is a function from A to B & *g* is another function defined from B to C, then mapping from A to C is known is composition of function.

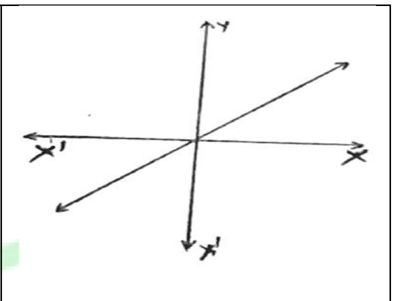
$$gof = g[f(x)]$$
$$fog = f[g(x)]$$

1. Explain constant, Identity, signum, step and exponential functions with their graphs.

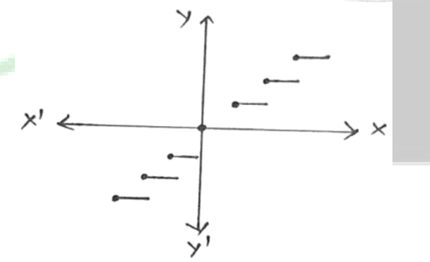
Ans.



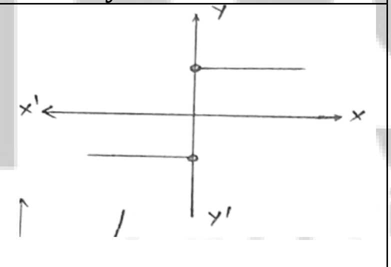
Constant function



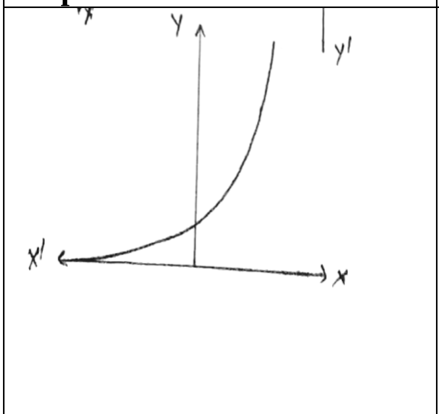
Identity function



Step function



Signum function

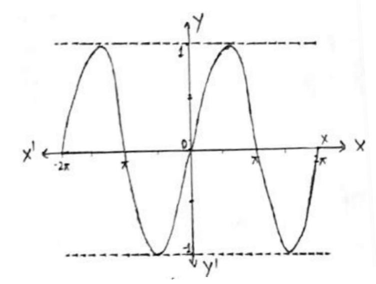
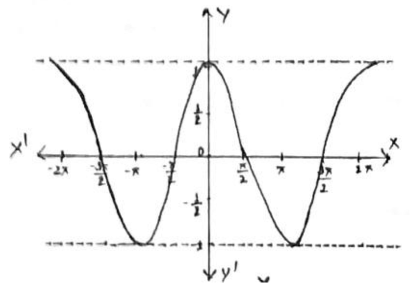
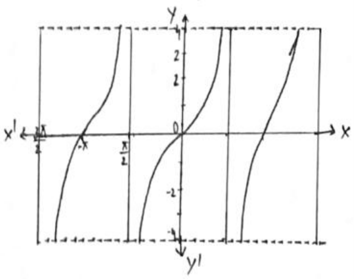
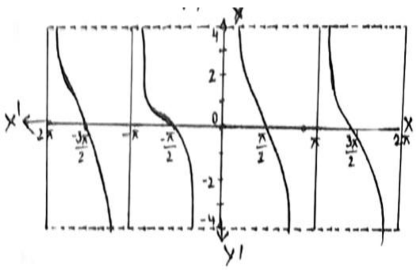
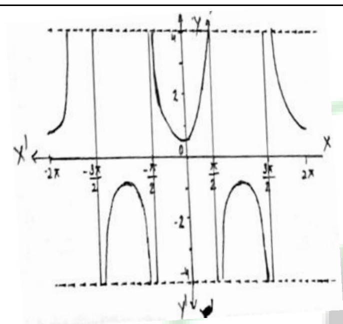
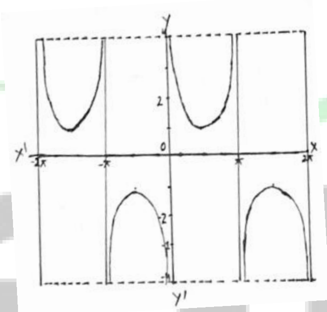
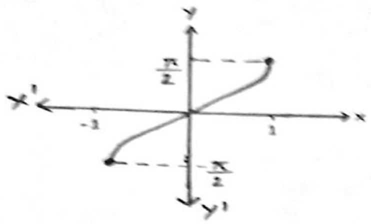
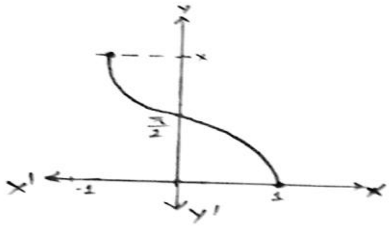
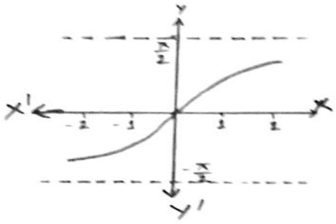
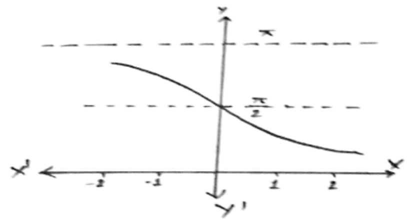


Exponential Function

Or

Explain all trigonometric and inverse trigonometric functions with their graphs.

Ans.

 *sin x*	 *cos x*
 *tan x*	 *cot x*
 *sec x*	 *cosec x*
 $sin^{-1}x$	 $cos^{-1}x$
 $tan^{-1}x$	 $cot^{-1}x$

Aims Tutorial
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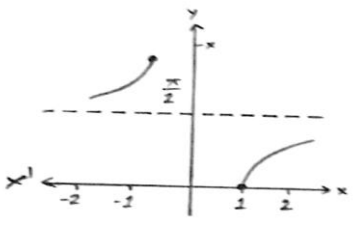
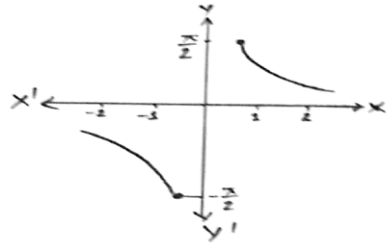
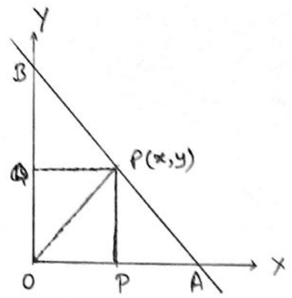
 $sec^{-1}\,x$	 $cosec^{-1}\,x$

2. Derive equation of straight line in slope form, in two point form, intercept form and on normal form.

Ans. Intercept Form: Let a line AB intersect the x-axis and y-axis at the point A and B respectively such that OA = a and OB = b. Now consider a point P (x, y) be any point on the line. Join OP draw perpendiculars PR and PQ from P on the x and y-axis respectively.



Area of right ΔOAB = area of ΔOPA + area of ΔOPB.

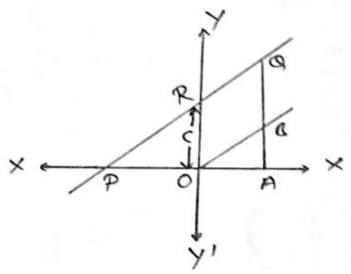
$\Rightarrow \quad \frac{1}{2} \times OA \times OB = \frac{1}{2} \times OA \times PR + \frac{1}{2}\ OB \times PQ$

$\Rightarrow \quad \frac{1}{2} \times a \times b = \frac{1}{2} \times a \times y + \frac{1}{2} \times b \times x$

So we obtain $\frac{x}{a} + \frac{y}{b} = 1$

Slope Intercept Form: Let's consider a line cutting intercept c on the y-axis making an angle θ with the x-axis.

Let Q(x, y) be any point on the line, now we draw from a perpendicular QA on x axis. Finally we draw a line parallel to PQ from origin which cut QA at B.



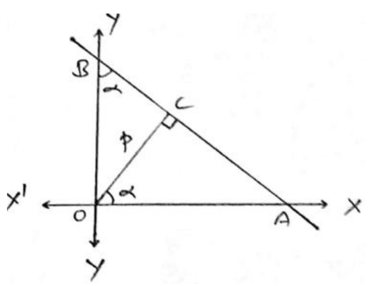
Now QA = QB + BA

$\Rightarrow$ QA = OR + OA tan θ

$\Rightarrow \quad y = x \tan\theta + c$

$\Rightarrow \quad y = mx + c$

Normal Form: Let's consider a line which cut at A and B at x axis and y axis. Length of the perpendicular from the origin form this line is p and the angle made by this perpendicular with the x-axis is α.



In right ΔOCA

$\cos\alpha = \frac{P}{OA} \Rightarrow OA = \frac{P}{\cos\alpha}$

In right ΔOBC

$\sin\alpha = \frac{P}{OB} \Rightarrow OB = \frac{P}{\sin\alpha}$

Now using the intercept

The equation is $\frac{x}{p/\cos\alpha} + \frac{y}{p/\sin\alpha} = 1$

$\Rightarrow \quad x\cos\alpha + y\sin\alpha = p$

Two point form: Consider two points (x_1, y_1) and (x_2, y_2). Now we want to find out a line passing through these points.

Slop intercept form is

$y = mx + c$ ………………….(1)

Since the line passes through these points

So, $y_1 = mx_1 + c$…………………(2)

and $y_2 = mx_2 + c$…………………(3)

now solving eq. (1) and (2)

$y - y_1 = m(x - x_1)$……………..(4)

subtracting eq. (2) and (3)

$y_2 - y_1 = m(x_2 - x_1)$

$\Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1}$

Now by equation (4)

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} = x - x_1$

Or

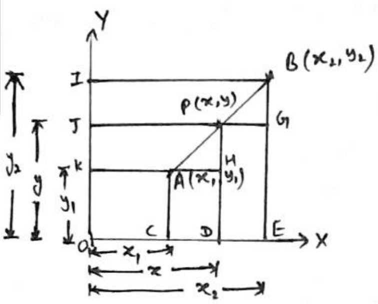
(a) State and prove section formulas.

Ans. A line segment AB can be divided in a given ratio m : n in two ways :

Internal Division: When point of division P lies between A and B then the division is called internal division.

If AB is the line segment with coordinate $A(x_1, y_1)$ & $B(x_2, y_2)$ be the given points & let P(x, y) be the point dividing AB internally in the ratio m: n, then the coordinate of P will be:

$$P(x,y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$



Proof: In ΔAPH and ΔPBG

∠G = ∠H (right angle)

∠A = ∠P (Corresponding angle)

ΔAPH ~ ΔPBG (triangles are similar)

$\frac{AH}{PG} = \frac{PH}{BG} = \frac{AP}{PB} = \frac{m}{n}$

And AH = CD = $x - x_1$,

PG = DE = $x_2 - x$,

$\frac{AH}{PG} = \frac{m}{n}$

$\Rightarrow \frac{x - x_1}{x_2 - x} = \frac{m}{n}$

$\Rightarrow n(x - x_1) = m(x_2 - x)$

$\Rightarrow x = \frac{mx_2 + nx_1}{m+n}$

Now

PH = JK = $y - y_1$,

BG = JI = $y_2 - y$

$\frac{PH}{BG} = \frac{m}{n} \Rightarrow \frac{y - y_1}{y_2 - y} = \frac{m}{n}$

$\Rightarrow n(y - y_1) = m(y_2 - y)$

$\Rightarrow y = \frac{my_2 + ny_1}{m+n}$

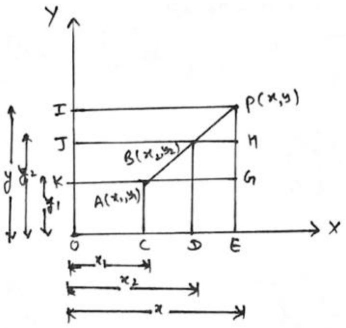
Hence, the coordinates of P are

$$P(x,y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

External Division: When point of division P lies outside i.e. in left or right side of AB then the division is called external division.

Let A (x_1, y_1) & B(x_2, y_2) be the given points & let P (x, y) be the point dividing AB externally in the ratio m: n. then the coordinate of P will be:

$$P(x,y) = \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$$



Proof: In ΔAPG and ΔPBH

∠G = ∠H (right angle)

∠A= ∠B (Corresponding angle)

ΔAPG ~ΔPBH (triangles are similar)

$\frac{AG}{BH} = \frac{PG}{PH} = \frac{AP}{PB} = \frac{m}{n}$

Since $\frac{AG}{BH} = \frac{m}{n}$

And AG = CE = x – x_1;

BH = DE = x – x_2;

Hence $\frac{AG}{BH} = \frac{m}{n}$

$\Rightarrow \frac{x-x_1}{x-x_2} = \frac{m}{n}$

$\Rightarrow$ n$(x - x_1)$ = m $(x - x_2)$

$\Rightarrow$ x = $\frac{mx_2 - nx_1}{m-n}$

Now PG = JK = y – y_1;

PH = JI = y – y_2

$\Rightarrow \frac{PG}{PH} = \frac{m}{n}$

$\Rightarrow \frac{y-y_1}{y-y_2} = \frac{m}{n}$

$\Rightarrow$ n $(y - y_1)$ = m $(y - y_2)$

$\Rightarrow$ y = $\frac{my_2 - ny_1}{m-n}$

Hence, the coordinates of p are

$$P(x,y) = \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$$