PERMUTATION

Any arrangement of a set of n object is known as permutation.

If there are n objects then a order of r different objects is called permutation of n object taken r at a time.

The total number of permutation of n objects taken r at a time denoted by ${}^nP_r$.

$${}^nP_r = n(n-1)(n-2)\ldots\ldots\ldots(n-r+1)$$

$${}^np_r = \frac{n!}{(n-r)!}$$

Properties of Permutation:

- ✓ ${}^np_n = n!$
- ✓ ${}^np_{n-1} = {}^np_n$

Permutation with Repetition of Objects:

The number of permutation of n objects of which p objects are of one type, q is of second type, r are of third type and remaining objects are different is, x =

$$\frac{n!}{p!\,q!\,r!}$$

Circular Permutation:

If objects are arranged in a circular order then the circular permutations of n different objects is $(n-1)!$

COMBINATION

A combination is a set or collections of objects where order does not matter.

In a combination the ordering of selected objects is immaterial whereas in a permutation, the ordering is essential.

- ✓ ${}^nC_r = \frac{n!}{(n-r)!\,r!}$

Relation between Permutation and Combination:

- ✓ ${}^np_r = r!\ {}^nC_r$

Properties of Combination:

- ✓ ${}^nC_n = {}^nC_{n-r}$
- ✓ ${}^nC_x = {}^nC_y \Rightarrow x = y \text{ or } x + y = n$

PROBABLITY

BASIC CONCEPTS AND DEFINITION

- **Experiment:** An action or operation which produce some well defined outcomes.
  There are two types of experiments:
  - ✓ Random
  - ✓ Deterministic

- **Random Experiment:** An experiment is said to be a random experiment, if its out-come can't be predicted with certainly.
  Eg: If a coin is tossed, we can't say, whether head or rail will appear. So it is a random experiment.

- **Deterministic Experiment:** An experiment is said to be a deterministic experiment, if its out-come can be predicted with certainly.
  Eg: The experiments that we conduct to verify the laws of science

- **Sample Space :** -The set of all possible out-comes of an experiment is called the sample – space.
  Eg: If we throw a dice, the number, that appears at top is any one of 1, 2, 3, 4, 5, 6,

  S = {1, 2, 3, 4, 5, 6} Similarly in the case of a coin, s = {H, T} and n (s) = 2.

- **Event:** Every subset of a sample space is an event. It is denoted by 'E'.
  Eg.: In throwing a dice S = {1, 2, 3, 4, 5, 6,}, the appearance of an even number will be the event E = {2, 4, 6}.

  Important types of Events:

- **Simple or elementary event:** An event, consisting of a single point is called a simple event.
  Eg: In throwing a dice s = {1, 2, 3, 4, 5, 6} so each of {1}, {2}, {3}, {4}, {5} and {6} is a simple event.

- **Compound or mixed event:** A subset of the sample space which has more than one element is called a mixed event.
  Eg: In throwing a dice, the event of odd numbers appearing is a mixed event, because E = {1, 3, 5}, which has '3' elements.

- **Equally likely events:** Events are said to be equally likely, if they have same chances of occurrence.
  Eg: When a dice is thrown, all the six-faces {1, 2, 3, 4, 5, 6,} are equally likely to come-up.
- **Exhaustive events:** When every possible outcome of an experiment is considered, the observation is called exhaustive events.
  Eg: When a dice is thrown, cases 1, 2, 3, 4, 5, 6 form an exhaustive set of events.

- **Mutually Exclusive or Disjoint Events:** If two or more events can't occur simultaneously, i.e. no two of them can occur together.

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Eg: When a coin is tossed, the event of occurrence of a head and the event of occurrence of a tail are mutually exclusive events.

- **Independent or Mutually independent Events:** Two or more events are said to be independent, if occurrence or non-occurrence of any of them does not affect the probability of occurrence or non-occurrence of other event.
  Eg: When a coin is tossed twice, the event of occurrence of head in the first throw and the event of occurrence of head in the second throw and independent events.

- **Difference between mutually exclusive and mutually Independent events:** Mutually exclusiveness is used, when the events are taken from the same experiment, whereas the independence is used, when the events are taken from different experiments.

- **Complement of an event :** let 'S' be the sample space for a random experiment, and 'E' be an event, then complement of 'E' is denoted by 'E^1' or E^c is the event [not E], i.e. the event that E does not occur.

PROBABILITY

Classical or Priori or Mathematical Definition of Probability:
Let an experiment result in *n* exhaustive, mutually exclusive and equally likely cases (events) of which *m* are favourable to the happening of an event *E* then the probability of happening event (E) is given by

$$p(E) = \frac{Number\ of\ favourable\ outcomes}{Number\ of\ total\ outcomes}$$

$$p(E) = \frac{m}{n}$$

Statistical or Empirical Definition of Probability:
Let and experiment be repeated a large number of times under the similar condition then the limiting value of $\frac{m}{n}$ where *m* is the favourable and *n* is the exhaustive cases is called the statistical probability.

$$p = \lim_{n\to\infty} \frac{m}{n}$$

Axiomatic Definition of Probability:
Let S be the sample space and A be any events the P(A) is called probability function.
If the following function properties or axioms are satisfied:

- ✓ The probability of an event ranges from 0 to 1. If the event cannot take place its probability shall be 0 and if it is certain i.e. $P(a) \leq 1$ and $P(a) \geq 0$.
- ✓ The probability of the entire sample space is 1 i.e. P(s) = 1.
- ✓ If A and B are mutually exclusive events then the probability of occurrence of either A and B denoted by

$$P(A \text{ or } B) = P(A) + P(B)$$

THEOREMS OF PROBABILITY

There are two important theorem of probability

- ✓ Addition theorem
- ✓ Multiplication theorem

Addition Theorem:

- **Statement:** If two events A and B are mutually exclusive the probability of occurrence of either A and B is the sum of the individual probability of A and B. i.e.
  - $P(A \text{ or } B) = P(A) + P(B)$
- **Proof:** Let event A can occur in a_1 ways and B in a_2 ways successfully. Let the total number of possibilities is n. Now the number of ways in which either event can occur is $a_1 + a_2$ .Then

$$P(A\ or\ B) = \frac{a_1 + a_2}{n}$$

$$= \frac{a_1}{n} + \frac{a_2}{n}$$

$$Here\ \ \frac{a_1}{n} = P(A)\ \ and$$

$$\frac{a_2}{n} = P(B)$$

$$P(A\ or\ B) = P(A) + P(B)$$

- When Events are not mutually exclusive then **Addition Theorem** can be written as:

$$P(A \text{ or } B) = P(A) + P(B) – P(A \text{ and } B)$$

Multiplication Theorem:

- **Statement:** If two events A and B are independent the probability that they both will occur is equal to the product of their individual probabilities. i.e.

$$P(A \text{ and } B) = P(A) \times P(B)$$

- **Proof:** Let event A can occur in n_1 ways and B in n_2 ways in which a_1 ways and a_2 ways are successful. Thus, the total number of successful happenings in both cases is $a_1 \times a_2$ .Then the total number of possible cases in $n_1 \times n_2$. Then

$$P(A\ and\ B) = \frac{a_1 \times a_2}{n_1 \times n_2}$$

$$= \frac{a_1}{n} \times \frac{a_2}{n}$$

$$Here\ \ \frac{a_1}{n} = P(A)\ \ and$$

$$\frac{a_2}{n} = P(B)$$

$$P(A\ and\ B) = P(A) \times P(B)$$

- When Events are dependent then **Multiplication Theorem** can be written as:

$$P(A\ and\ B) = P(A) \times P\left(\frac{B}{A}\right)$$

CONDITIONAL PROBABILITY

Let two event A and B are said to be dependent when B can occur only when A is known to have occurred is known as conditional probability is denoted by $P\left(\frac{B}{A}\right)$

$$P\left(\frac{B}{A}\right) = \frac{P(AB)}{P(A)}$$

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